13. a) 5×5
The base is 5. There are 2 equal factors, so the exponent is 2.
So, $5 \times 5 = 5^2$
$= 25$

b) $3 \times 3 \times 3 \times 3$
The base is 3. There are 4 equal factors, so the exponent is 4.
So, $3 \times 3 \times 3 \times 3 = 3^4$
$= 81$

c) $10 \times 10 \times 10 \times 10 \times 10$
The base is 10. There are 5 equal factors, so the exponent is 5.
So, $10 \times 10 \times 10 \times 10 \times 10 = 10^5$
$= 100\ 000$

d) $-(9 \times 9 \times 9)$
The base is 9. There are 3 equal factors, so the exponent is 3.
So, $-(9 \times 9 \times 9) = -9^3$
$= -729$

e) $(-2)(-2)(-2)$
The base is (-2). There are 3 equal factors, so the exponent is 3.
So, $(-2)(-2)(-2) = (-2)^3$
$= -8$

f) $-(-4)(-4)(-4)$
the base is (-4). There are 3 equal factors, so the exponent is 3.
So, $-(-4)(-4)(-4) = -(-4)^3$
$= 64$

g) $(-5)(-5)(-5)(-5)$
The base is (-5). There are 4 equal factors, so the exponent is 4.
So, $(-5)(-5)(-5)(-5) = (-5)^4$
$= 625$

h) $-(5)(5)(5)(5)$
The base is 5. There are 4 equal factors, so the exponent is 4.
So, $-(5)(5)(5)(5) = -5^4$
$= -625$

i) $-(-5)(-5)(-5)(-5)$
The base is (-5). There are 4 equal factors, so the exponent is 4.
So, $-(-5)(-5)(-5)(-5) = -(-5)^4$
$= -625$

14. a) I predict the answer is positive, because the base is positive.
$2^3 = 2 \times 2 \times 2$
$= 8$

b) I predict the answer is positive, because the base is positive.
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 1\ 000\ 000$

c) I predict the answer is positive, because the base is positive.
$3^1 = 3$

d) The exponent applies only to the base 7, and not to the negative sign.
I predict the answer is negative, because there is a negative sign in front of the base.
$-7^3 = -(7^3)$
$= -(7 \times 7 \times 7)$
$= -343$

e) The sign of a product with an odd number of negative factors is negative.
So, I predict the answer is negative, because the base is negative, and the exponent is an odd number.
$(-7)^3 = (-7)(-7)(-7)$
$= -343$

f) The sign of a product with an even number of negative factors is positive.
So, I predict the answer is positive, because the exponent is an even number.
$(-2)^8 = (-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2)$
$= 256$

g) The exponent applies only to the base 2, and not to the negative sign.
So, I predict the answer is negative, because there is a negative sign in front of the base.
$-2^8 = -(2^8)$
$= -(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$
$= -256$

h) The exponent applies only to the base 6, and not to the negative sign.
So, I predict the answer is negative, because there is a negative sign in front of the base.
$-6^4 = -(6^4)$
$= -(6 \times 6 \times 6 \times 6)$
$= -1296$

i) The sign of a product with an even number of negative factors is positive.
So, I predict the answer is positive, because the exponent is an even number.
$(-6)^4 = (-6)(-6)(-6)(-6)$
$= 1296$

j) The sign of a product with an even number of negative factors is positive.
But, there is another negative sign in front of the base, so I predict the answer is negative.
$-(-6)^4 = -((-6)(-6)(-6)(-6))$
$= -(1296)$
$= -1296$

k) The sign of a product with an odd number of negative factors is negative.
So, I predict the answer is negative, because the base is negative, and the exponent is an odd number.
$(-5)^3 = (-5)(-5)(-5)$
$= -125$

l) The exponent applies only to the base 4, and not to the negative sign.
So, I predict the answer is negative, because there is a negative sign in front of the base.
$-4^4 = -(4^4)$
$= -(4 \times 4 \times 4 \times 4)$
$= -256$

16. a) $3^{12} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 531\ 441$

b) $-7^7 = -(7^7)$
$= -(7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7)$
$= -823\ 543$

c) $5^{11} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 48\ 828\ 125$

d) $-(-4)^{10} = -((-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4))$
$= -(1\ 048\ 576)$
$= -1\ 048\ 576$

e) $(-9)^8 = (-9)(-9)(-9)(-9)(-9)(-9)(-9)(-9)$
$= 43\ 046\ 721$

f) $2^{23} = 2 \times 2$
$= 8\ 388\ 608$

17. a) i) $4^3 = 4 \times 4 \times 4$ As repeated multiplication
$= 64$ Standard form

ii) $-4^3 = -(4^3)$
$= -(4 \times 4 \times 4)$ As repeated multiplication
$= -64$ Standard form

iii) $-(-4^3) = -(-(4^3))$
$= -(-(4 \times 4 \times 4))$ As repeated multiplication
$= -(-64)$ $-(-64)$ is the opposite of -64, which is 64
$= 64$ Standard form

iv) $(-4^3) = (-(4^3))$
$= (-(4 \times 4 \times 4))$ As repeated multiplication
$= (-64)$
$= -64$ Standard form

b) 4^3 is positive because the base 4 is positive.
$-(-4^3)$ is positive because -4^3 is negative, and $-(-4^3)$ is the opposite of -4^3.

-4^3 is negative because the negative sign is not part of the base of the power.
(-4^3) is negative because the exponent is inside the brackets and has no effect on the negative sign.

c) i) $4^2 = 4 \times 4$ As repeated multiplication
$= 16$ Standard form

ii) $-4^2 = -(4^2)$
$= -(4 \times 4)$ As repeated multiplication
$= -16$ Standard form

iii) $-(-4^2) = -(-(4^2))$
$= -(-(4 \times 4))$ As repeated multiplication
$= -(-16)$ $-(-16)$ is the opposite of -16, which is 16
$= 16$ Standard form

iv) $(-4^2) = (-(4 \times 4))$ As repeated multiplication
$= -16$ Standard form

d) 4^2 is positive because the base 4 is positive.
$-(-4^2)$ is positive because -4^2 is negative, and $-(-4^2)$ is the opposite of -4^2.

-4^2 is negative because the negative sign is not part of the base of the power.
(-4^2) is negative because the exponent is inside the brackets and has no effect on the negative sign.

e) 5^4, -5^4, $-(-5^4)$, (-5^4)
9^1, -9^1, $-(-9^1)$, (-9^1)
10^9, -10^9, $-(-10^9)$, (-10^9)
If a power has a positive base and the power does not have a negative sign in front of it, the product will be positive.
If the power has a positive base, and there are two negative signs in front of the power, the product will be positive because one negative sign produces a negative product, then the second negative sign indicates that the product will be the opposite number.
If there is a negative sign in front of the power and the power has a positive base, the product will be negative.

18. a) All three expressions are equal.
For -3^5, the negative sign is not part of the base of the power:
$-3^5 = -(3^5)$
$= -(3 \times 3 \times 3 \times 3 \times 3)$
$= -243$
For $(-3)^5$, the brackets indicate that the negative sign is part of the base of the power. The product is negative because the exponent is odd, and the product of an odd number of negative integers is negative:
$(-3)^5 = (-3)(-3)(-3)(-3)(-3)$
$= -243$
For (-3^5), the brackets serve no purpose and (-3^5) is the same as -3^5.
So, $(-3^5) = -(3^5)$
$= -(3 \times 3 \times 3 \times 3 \times 3)$
$= -243$

b) The expressions -4^6 and (-4^6) are equal.
For -4^6, the negative sign is not part of the base of the power:
$-4^6 = -(4^6)$
$= -(4 \times 4 \times 4 \times 4 \times 4 \times 4)$
$= -4096$
For (-4^6), the brackets serve no purpose and (-4^6) means the same as -4^6.
So, $(-4^6) = -(4^6)$
$= -(4 \times 4 \times 4 \times 4 \times 4 \times 4)$
$= -4096$
For $(-4)^6$, the brackets indicate that the negative sign is part of the base of the power. The product is positive because the exponent is even, and the product of an even number of negative integers is positive:
$(-4)^6 = (-4)(-4)(-4)(-4)(-4)(-4)$
$= 4096$

20. a) $4 = 2 \times 2$
$= 2^2$

b) $16 = 2 \times 2 \times 2 \times 2$
$= 2^4$

c) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

d) $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^8$

e) $32 = 2 \times 2 \times 2 \times 2 \times 2$
$= 2^5$

f) $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^7$
I multiplied 2 by 2 repeatedly until I reached the number. I could have also divided the number by 2 repeatedly and counted how many times I had to do this until I reached 1.

21. I tried dividing each number by 2, by 3, by 4, and so on.
I know that 2, 4, 8, and 16 are all multiples of 2, so when I wrote the power with a base of 2, I then checked to see if it could be written with a different base that is a multiple of 2.
When the exponent of a power is even and greater than 2, it can be written a different way with a base that is the square of the current base.
The same is true for 9 as a multiple of 3.
a) i) $16 = 16^1$
$16 = 4 \times 4$
$= 4^2$
$16 = 2 \times 2 \times 2 \times 2$
$= 2^4$

ii) $81 = 81^1$
$81 = 9 \times 9$

$= 9^2$
$81 = 3 \times 3 \times 3 \times 3$
$= 3^4$

iii) $256 = 256^1$
$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^8$
$256 = 4 \times 4 \times 4 \times 4$
$= 4^4$
$256 = 16 \times 16$
$= 16^2$

b) To find more numbers that can be written as a power in more than one way, I chose a number, squared it, then squared the product again.
I chose 10, squared it to get 10 × 10 = 100, then squared 100 to get 100 × 100 = 10 000.
So, one number that can be written as a power in more than one way is 10 000 because:
$10\,000 = 100^2$
$= 100 \times 100$
and also:
$10\,000 = 10^4$
$= 10 \times 10 \times 10 \times 10$

I chose 5, squared it to get 5 × 5 = 25, then squared 25 to get 25 × 25 = 625.
So, another number that can be written as a power in more than one way is 625 because:
$625 = 25^2$
$= 25 \times 25$
and also:
$625 = 5^4$
$= 5 \times 5 \times 5 \times 5$

22. a) The powers in each pair have the same numbers in them.
The powers are different because the bases and the exponents are interchanged.

b) i) 3^2 is greater than 2^3 because $3^2 = 9$ and $2^3 = 8$

ii) 2^5 is greater than 5^2 because $2^5 = 32$ and $5^2 = 25$

iii) 3^4 is greater than 4^3 because $3^4 = 81$ and $4^3 = 64$

iv) 4^5 is greater than 5^4 because $4^5 = 1024$ and $5^4 = 625$

23. I know that 3^5 is greater than 3^4, because the powers have the same base, but 3^5 has a greater exponent.
I know that 6^3 is greater than 5^2 because 6^3 has a greater base and a greater exponent than 5^2.
3^5 can be written as $3 \times 3 \times 3 \times 3 \times 3$
6^3 can be written as $6 \times 6 \times 6 = 3 \times 3 \times 3 \times 2 \times 2 \times 2$
3^4 can be written as $3 \times 3 \times 3 \times 3$
The product of the first three factors in each repeated multiplication is the same, so look at the remaining factors.
$3 \times 3 = 9$, and $2 \times 2 \times 2 = 8$; $9 > 8$, so $3^5 > 6^3$
The remaining factor in 3^4 is 3, so $6^3 > 3^4$
And $3^4 > 5^2$ since $3 \times 3 \times 3 \times 3 = 81$, $5 \times 5 = 25$ and $81 > 25$.
So, from greatest to least:
$3^5, 6^3, 3^4, 5^2$

24. a) There are 64 1-unit squares on the checkerboard.
$64 = 8 \times 8$
$= 8^2$

b) There are 49 2-unit squares on the checkerboard.
$49 = 7 \times 7$
$= 7^2$

c) There are 36 3-unit squares on the checkerboard.
$36 = 6 \times 6$
$= 6^2$

d) There are 25 4-unit squares on the checkerboard.
$25 = 5 \times 5$
$= 5^2$

e) There are 16 5-unit squares on the checkerboard.
$16 = 4 \times 4$
$= 4^2$

f) There are nine 6-unit squares on the checkerboard.
$9 = 3 \times 3$
$= 3^2$

g) There are four 7-unit squares on the checkerboard.
$4 = 2 \times 2$
$= 2^2$

h) There is one 8-unit square on the checkerboard.
$1 = 1 \times 1$
$= 1^2$

Each number of squares is a square number that decreases as the size of the square increases.

25. You can tell if a number is a square number by trying to determine if there is a number that, when multiplied by itself, has a product equal to the number being investigated.
For example, 2096; try $40 \times 40 = 1600$ and $50 \times 50 = 2500$, determine $44 \times 44 = 1936$, then $46 \times 46 = 2116$, so 2096 is not a square number because 45×45 would have a product with 5 in the ones place. A similar method works for a cube number, but multiply a number by itself twice. For example, 2096; try $10 \times 10 \times 10 = 1000$, and $12 \times 12 \times 12 = 1728$, and $13 \times 13 \times 13 = 2197$, so 2096 is not a cube number either.